

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT2

Branch: B. Tech (All)

Semester : 1

Date : 14/03/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) If $y = \sin 2x \cos 2x$ then y_n equal to
 (A) $\frac{1}{2}(4)^n \cos\left(\frac{n\pi}{2} + 4x\right)$ (B) $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 4x\right)$
 (C) $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 2x\right)$ (D) none of these
- b) nth derivative of $y = \frac{1}{x+a}$ is
 (A) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ (B) $\frac{(-1)^{n-1} n!}{(x+a)^{n+1}}$ (C) $\frac{(-1)^n n!}{(x+a)^n}$ (D) none of these
- c) If $1+y = e^{-x}$ or $y = e^{-x} - 1$, then x equal to
 (A) $-y + \frac{y^2}{2} - \frac{y^3}{3} + \frac{y^4}{4} + \dots$ (B) $y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \dots$
 (C) $y + \frac{y^2}{2} + \frac{y^3}{3} + \frac{y^4}{4} + \dots$ (D) None of these
- d) The series $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$ represent expansion of
 (A) $\sin x$ (B) $\cos x$ (C) $\sinh x$ (D) $\cosh x$
- e) $\lim_{x \rightarrow \infty} x^n e^{-ax}$ (n being a positive integer and $a > 0$) = _____
 (A) -1 (B) 0 (C) 1 (D) None of these
- f) $\lim_{x \rightarrow \infty} x \left(a^{\frac{1}{x}} - 1 \right) =$ _____
 (A) $\log_e a$ (B) 0 (C) 1 (D) none of these
- g) If $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)}$ is equal to
 (A) 1 (B) -1 (C) zero (D) none of these



- h) If $u = f\left(\frac{x}{y}\right)$ then
 (A) $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = 0$ (B) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$ (C) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u$ (D) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$
- i) If $Q = r \cot \theta$, then $\frac{\partial Q}{\partial r}$ is equal to
 (A) $\cot \theta$ (B) $-\cos^2 \theta$ (C) $\cot \theta - r \operatorname{cosec}^2 \theta$ (D) $\frac{1}{2} \cot \theta$
- j) If $u = y^x$, then $\frac{\partial u}{\partial x}$ is
 (A) xy^{x-1} (B) 0 (C) $y^x \log x$ (D) none of these
- k) If $1, \alpha_1, \alpha_2, \dots, \alpha_{n-1}$ are the n roots of unity, then $(1 + \alpha_1)(1 + \alpha_2) \dots (1 + \alpha_{n-1})$ is equal to
 (A) $n-1$ (B) n (C) -1 (D) none of these
- l) If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then
 (A) $a = 2, b = -1$ (B) $a = 1, b = 0$ (C) $a = 0, b = 1$ (D) $a = -1, b = 2$
- m) An $n \times n$ homogeneous system of equations $AX = 0$ is given. The rank of A is $r < n$. Then the system has
 (A) $n-r$ independent solutions (B) r independent solutions (C) no solution (D) n independent solutions
- n) The rank of the diagonal matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is
 (A) 1 (B) 2 (C) 3 (D) -2

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) If $y = \frac{x^4}{(x-1)(x-2)}$ then find y_n . (5)

b) Expand $e^{\sin x}$ as a series of ascending power of x upto x^4 . (5)

c) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = -\frac{4}{(x+y)^2}$ (4)

Q-3 Attempt all questions (14)

a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that (5)

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0.$$



b) Prove that $(1+x)^x = 1+x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$ (5)

c) Evaluate: $\lim_{x \rightarrow \frac{\pi}{4}} (1 - \tan x) \sec 2x$ (4)

Q-4 Attempt all questions (14)

a) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^{1-\cos x}$ (5)

b) If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$ then find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$. (5)

c) Expand $\log x$ in powers of $(x-2)$. (4)

Q-5 Attempt all questions (14)

a) If $u = \sec^{-1} \left(\frac{x^2 + y^2}{x-y}\right)$ then prove that (5)

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\cot u (\cot^2 u + 2).$$

b) Evaluate: $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$ (5)

c) If $y = \cos x \cos 2x \cos 3x$ then find y_n . (4)

Q-6 Attempt all questions (14)

a) The power consumed in an electric resistor is given by $P = \frac{E^2}{R}$ (in watts). If $E = 200$ volts and $R = 8$ ohms, by how much does the power change if E is decreased by 5 volts and R is decreased by 0.20 ohms? (5)

b) Prove that $\left(\frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i \sin\left(\frac{n\pi}{2} - n\theta\right)$ (5)

c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. (4)

Q-7 Attempt all questions (14)

a) Reduce the matrix $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$ to the normal form and find its rank. (5)

b) Find the continued product of all the values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$. (5)

c) Prove that $\sec h^{-1}(\sin \theta) = \log \cot \frac{\theta}{2}$. (4)

Q-8 Attempt all questions (14)

a) Examine whether the following equations are consistent and solve them if they are consistent. (5)

$$2x + 6y + 11 = 0, \quad 6x + 20y - 6z + 3 = 0, \quad 6y - 18z + 1 = 0$$



b) Find the fourth roots of unity and sketch them on the unit circle. **(5)**

c) Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. **(4)**

